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Relational Classification in
Behavior Analysis and Ecological Psychology

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Theoretical Section

Structure and Function in Behavior Analysis
Abstract

The distinction between structure and function is central to behavior-analytic thinking. It can be connected to ideas of selection, used to contrast behavior analysis with other disciplines, and to describe independent inquiry. This paper endeavors to technically define the structure-function distinction as data, and so render it empirically accountable. The philosophical concept of multiple realizability is used to do so. It is argued that structure-function relationships can be explicated as operationalized multiple realizability. Three criteria for structure-function relationships are distilled. First, structural variables $s_1...s_n$ must converge on the same relational variable(s) $C_1...C_n$. Second, the structural variables $s_1...s_n$ must divergence from each other by criteria $D_1...D_n$. Third, the structural variables $s_1...s_n$ must be relevant for instantiating the relational variables(s) $C_1...C_n$. It is the hope that this explication will be a move towards a conception of structure-function relations that is amendable to experimental analysis.

Keywords: structure, function, behaviorism, multiple realizability.
Structure and Function in Behavior Analysis

Behavior analysts classify behavior in terms of its function. That is, behavior is a unit insofar as it enters orderly relations with the environment. This concept grounds the very meaning of recurrence in behavior-analytic data. Introductory text books are quick to present the notion of function in some form or other (Catania, 2013; Cheney & Pierce, 2013; Cooper, Heron, & Heward, 2007).

One issue on this subject is how to demarcate functions across levels. The famous example is Skinner’s (1981) partitioning of selection into three levels. This is a theoretical work on the nature of such demarcation with focus on functional level-hood. The paper starts with the premise that level-hood is a partnership between continuity and autonomy. First, the higher-level is continuous with the lower-level. That is, the higher-level constitutively entails (Glenn, 2003), logically implicates, ontologically depends on (Schnaitter, 1999), is abstracted from (Hayakawa, 1964), or is in contact with (Houmanfar, Rodrigues, & Ward, 2010) the lower-level. Second, the higher-level is autonomous from the lower-level. The higher-level properties are qualitatively distinct (Houmanfar et al., 2010), differently categorized (Holth, 2001), pragmatically disparate (Simon, 1962), or epistemically independent (Viklund, 2016).

The behavior-analytic distinction between structure and function is reminiscent of this idea (Catania, 1973b). Skinner (1981) believed that the selection of behavior can be investigated in its own right, and yet be continuous with selection on higher and lower levels. Catania argued that behavior can be investigated from a structural and functional perspective, and that although behavior is accompanied by both, structure and function are orthogonal. This article will therefore present functional level-hood as a technical extension of the structure-function distinction, and so build on already established behavior-analytic terminology. Function is considered the higher level, and structure the lower level. Function is autonomous from structure and yet continuous with it.
The structure-function distinction is extended by the philosophical method of explication. To explicate a concept means to render it technically perspicuous in a way that respects previous usage (Gupta, 2008). All previous usage cannot be accommodated. If the concept has been ambiguously employed, it is in the nature of explication to generate some outliers. Also, this is not a review of all literature on structure-function usages. This paper aims to explicate the structure-function distinction specifically for the purpose of (a) paying homage to the notion of functional level-hood and (b) render the distinction empirically accountable.

The need for empirical accountability requires some explanation. Note that none of the above listed versions of continuity and autonomy are empirical. They are highly theoretical interpretations. This is not necessarily a problem, but consider the following example.

The structure-function distinction as interpreted above, along with selection, was mentioned in the same breath by the philosopher Rosenberg (2001): “It is the nature of any mechanism which selects for effects that it cannot discriminate differing structures with identical effects”, and “selection for function is blind to differences in structure […]]” (p.737). This is similar to Catania’s (1973b) claim that structure and function are orthogonal, but with the additional assumption that selection is a source of orthogonality.

There are two interrelated problems with this claim. First, the distinction between selection as process and procedure creates a fork in the road for Rosenberg. As a procedure, the claim is true by definition. If we describe selection as just consequences that are indiscriminately contingent upon a variety of behaviors, then all we have is a description of a procedure and not a phenomenon. As a process, the claim is empirical and yet to be proved. Second, a new problem arises if we choose the empirical interpretation. The empirical question is undefined – there is no clear evidential standard. Treatments like Catania (1973b)
lack the precision and clarity needed to empirically evaluate the proposition that selection – as a process – targets function indiscriminately of structure. To understand this, one must let go of the idea that structure-function relations are obvious. Catania appeals to intuitive cases, such as functional versus formal treatments of language. Such case studies may be helpful, but they are not sufficient for a general understanding.

Despite being a core aspect of the behavior-analytic approach, there is currently no technical definition that specifies empirical requirements for structure-function relationships. Such specification is normally valued by behavior analysts (Baer & et al., 1987; Baer, Wolf, & Risley, 1968). Rosenberg’s claim exemplifies how such a definition can be helpful in studies of selection and functional level- hood as just outlined.

This article proceeds to provide an explication of structure-function relations against this background. It will outline the nuanced and multi-faceted ways that structure-function relations can be understood, along with the many decisions that must be made to frame these in a meaningful way. It will be argued that structure versus function is not a classificatory or qualitative distinction. Rather, structure-function relations are quantifiable empirical facts. These facts are meaningful for experimental control, and for that reason informative of when it may be interesting to speak of levels in a prediction and control paradigm. Even if none of the positive claims are accepted, this article will have achieved half its purpose if it demonstrates that structure-function relations cannot be taken for granted.

The explication project is divided into three parts. The first suggests the essential components of structure and function. The second provides a theoretical interpretation of how these components ought to be configured. The third part outlines the suggested explication. The article concludes by discussing anticipated critiques, unresolved issues, and related research.
The Components of Structure and Function

The purpose of this section is to clarify and limit the scope of the structure-function distinction. The full definition is not offered here, only the necessary components. That is, the defining features of structure and function. Here I discuss what to remove, what to keep, and why.

Relationality

Function in behavior analysis can be identified as doing, and doing can more narrowly be identified as relationality (Morris, 1988). Relationality has previously been discussed in the context of specific units such as operants (e.g., Catania, 1973a; Glenn, Ellis, & Greenspoon, 1992; Lee, 1992) or in terms of functional kinds such as selection (Skinner, 1981). None of this matter here, because function is not necessarily tied to selection as a causal mode or a specific unit. All that matter is the core idea of a functional relation between a dependent and independent variable (Chiesa, 1994). Functional relations are causal in the sparse sense that, unlike correlational knowledge, causal knowledge entails knowing the outcome of manipulation (Pearl, 2000). In this sparse sense, functional- and causal relations are interchangeable.

Functional relations serve a taxonomic role in behavior analysis. Skinner (1935) proposed that we individuate units of analysis as relata. Relata are the properties that enter into a relation. For example, we employ one-to-one correlations of stimuli and responses to group together all heterogenous properties that satsify this correlation. Stimuli and responses are hence relata for the reflex relation. Light at different intensities is relata for the reflex relation when any instance of that range makes the pupils contract. This is top-down classification, meaning that the relation is the method of grouping properties. That places the structures in a different causal niche than the functional units. The researcher is not directly committed to the replication of structures when asserting a function because top-down
classification preserves the generic identity of the relation despite divergence in relata properties; see also Skinner (1938). We speak of reinforcers instead of food to allow other relata to act as reinforcers, and we speak of discriminative stimuli to allow a variety of energy changes to act as discriminative stimuli. This idea of functional relations – in the context of experimental control – will serve as the conceptual foundation for everything that will be said henceforth.

**Autonomy**

Functional classes have been shown to persist through divergence in structure across experiments, across subjects, and species (Catania, 2013; Cheney & Pierce, 2013). Let us call this “substitutability”, referring to when divergent structures converge on the same function. Two points will be made. First, substitutability is empirical. This follows from functional units being individuated specifically by causal relations. Causal relations do not stay the same in the face of structural divergence merely by definition. We can discover that some placeholders are different than others. The test for sameness is whether the properties of the relata can instantiate the same causal relation. For example, classical conditioning relations are not the same along all dimensions regardless of the placeholder used (e.g., Garcia & Koelling, 1966). For example, some relata facilitate conditioning more than others. Or to put the point more generally, for the law of effect to hold across species, there must be empirical substitutability because rats and pigeons necessarily employ different structures.

Second, substitutability is a type of autonomy. Think here of autonomy as a non-confound. If functional relations remain invariant in the face of structural change then, in that sense, they are autonomous from structure. Structure can be conceptualized as a non-confound. For example, the structural differences in how a rat presses a lever are not necessarily a confound for its schedule performance. Or, the structural differences of discriminative stimuli used to signal reinforcement is not necessarily a confound for the
resulting probabilistic relation between behavior and the stimuli. If any of these differences were confounds for these relations, they would effectively be on the same functional levelhood as far as experimental control is concerned. For the function to be on a different level from structure, the latter must not confound the former. This is autonomy in the following sense.

Presumably, every variation in structure has a cause and substitutability means \textit{experimental freedom} from such causes. Structural variation is not a confound when substitutability obtains, otherwise variation in structure should negate or alter the functional relation (e.g., Garcia & Koelling, 1966). Also, if there is no substitutability, the relation is empirically locked to specific properties, meaning that only these properties can serve as relata. The attempt to use different relata will not allow the specific relation to form. Hence, although structure alone does not determine relationality, structural substitutability determines the nomothetic range over which relations extend, and therefore how we must categorize relations for purposes of prediction and control. For example, imagine having plenty of money in a village with a severely limited purchasing scope – all you can buy is coffee. We have the relation of monetary value, along with placeholders (or relata): Pieces of paper and coffee. The paper may take on the functional role of money, but that role does not extend over anything else than coffee. The one that might now be inclined to redefine the putative money as coffee coupons have re-categorized the relation on the basis of substitutability. The rationale for this can be construed as follows. When only one type of relata can instantiate the relation, the two are epistemologically indistinguishable – the relata is an index for the relation and vice versa. To put the point in terms of methodology, the narrower the range of divergent relata that can instantiate the same relation, the more the relata properties matter for the experimental control of that relation. Likewise, the more structural substitutability, the more can the relations be investigated in their own right.
Continuity

Three points will be made on continuity. First, one should distinguish between structure and topography. “Structure” will henceforth mean constitutively relevant features of functional classes. “Topography” will mean features that accompany yet are not constitutive of the function. The human appendix accompanies much operant behavior but it can be removed without negating such behavior. Speech modalities such as vocal cords or hand gestures are structure for verbal behavior (e.g., Tincani, Bondy, & Crozier, 2011). They are structure because a verbal class, such as manding, requires a modality for any emitted instance of that class. That requirement is on the instance and not the class. The word “necessary” may incorrectly suggest that one is talking about the class, so this article settles for the term “relevance”.\(^1\) Compare this with topography. A dog may approach us if we speak to it in a certain tone. If the tone is all that is required, it follows that the tone-approach relation can be instantiated without the grammar of any human language. The presence of grammatical language is merely topographical to the tone-approach relation. In sum, topographical variables can both vary and be set to zero without negating the functional relation; likewise, structural variables can vary but they cannot be set to zero.

The relevance criterion separates substitutability into two versions, one trivial and one interesting. It was said above that substantiality is empirical and entails a form of autonomy. There is a trivial version of this, which is that there are many events in the world that can change without being confounds. The weather on Mars can vary and it does not confound our experiments. This is a trivial form of substitutability, because our experiments would not be confounded even if Mars did not exist. To be interesting, the substitutability in question must somehow be constitutive of the process we claim it is not a confound for. Unlike the weather

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\(^1\) This derives from Mackie’s (1965) notion of an INUS condition, and Gillet’s (2003) ideas about multiple realization. See the section on structure and function as multiple realizability.
on Mars, certain biomechanical movements are necessary to perform on a reinforcement schedule. Anything that interferes with all biomechanical movement will certainly be a confound for schedule performance. The purpose of a relevance criterion is to cut out the trivial cases of substitutability and preserve the interesting ones.

Second, we should ask what is meant by “structure”. Catania (2013) was prepared to call any organizational property structure whether physical or formal, such as phonemes and syntax. Or structure is defined as the form of the behavior (Cheney & Pierce, 2013), which appear synonymous with either structure or topography (e.g., Jones & Carr, 2004; Morris, 1988). Patterned features of biomechanical movement and neurological facts can also be included (Donahoe & Palmer, 1994).

At issue is whether structure is independent of function, or if structure exists merely relative to function. Here are two reasons to deny that structure is an independent concept. First, consider the idea that structure has organization or form. The problem seems to be that we also wish to say that function has organization and form. So, there is a choice to be made. Either we say that both structure and function has organization or form, or we say that organization and form are relative aspects of function – which we might call “structure”. Second, consider the lumping together of formal and physical properties – say, predicate calculus and speech modalities – under the heading of “structure”. If structure is independent, we need to state what formal and physical properties have in common, excluding the trivial commonality of not being part of the function. This problem goes away if we accept that structure is merely the formal and physical properties we pick out relative to a given function.

This idea parallels the thought tradition of Dewey (1896), Skinner (1935), and Catania (1973a) on stimulus and response as co-defining units. Structure and function can be treated the same way. We can individuate structure by its relative position to function. Structure
names the properties of the relata whether those properties be formal or material. This means that structure can be lower-order functional relations. For example, environment-individual relations can be structure for environment-group relations. A particular usage of structure-function relations drops out when we grant that structure is a relative concept. Behavior analysts have used the distinction to contrast their approach with other disciplines (e.g., Catania, 1973b, 2013; Cheney & Pierce, 2013). This is question begging if structure is a relative concept, because to agree on what is structure one must agree on what is function. If other disciplines do not recognize the same functional categories, or none at all, then “structure” ceases to mean anything.

Third, the structure-function distinction can be explicated as phenomena or data. Phenomena are the processes we claim to be studying and data are the idiosyncratic observables serving as evidence for these processes (Bogen & Woodward, 1988). We study the data from lever presses by rats to discover operant phenomena. Data is idiosyncratic and indirect information about the phenomena, in the practical sense that operants are not necessarily lever presses and lever presses are not necessarily operant behavior. This paper limits itself to structure and function as data.

In sum, three components of the structure-function distinction stand out: (a) function as causal relationality, (b) substitutability of structure, and (c) relevance of structure to function. This paper takes (a) for granted, and attempts to contribute to (b) and (c) since these pertain to autonomy and continuity.

Multiple Realizability

The previous section outlined the components of structure and function. This section will suggest how they should be configured. Recall that top-down classification – from relation to relata – hinges on the empirical fact that divergent structures converge on the same function. Behavior analysts and philosophers alike have taken an interest in such
convergence-divergence relations, and molded these into various irreducibility arguments. Three examples from behavior analysis will be listed, showing that these ideas are not novel to the discipline. Their common form offers a way to conceptualize structure-function relationships. All three rely upon the same argument: irreducibility through *multiple realizability* (Bickle, 2008), although the authors seem unaware of the term. Multiple realizability has been described as a “one-to-many” relation (Brigandt & Love, 2008) or “sameness through difference” (Funkhouser, 2007, p. 468).

**Respondent Behavior**

Schaal (2003) discusses a study of two species of mollusks in which researchers aimed to reduce the mollusks’ respondent behavior to neurological mechanisms. Both mollusks adhered to the same rules of respondent conditioning with only minor discrepancies. In this sense, they were "behaviorally homogenous" (p.91). The neurological reduction was successful, but it did not preserve the homogeneity found on the behavioral level. Schaal points to findings indicating that these two mollusk species obeyed the same rules of respondent conditioning for neurologically different reasons. On this level, the two mollusks were "neurologically heterogeneous" (p.91). The neurological level of analysis also accounted for minor discrepancies in response patterns; that is, the neurological heterogeneity ‘trickled up’, or asserted itself, on the behavioral level in a noticeable yet negligible way. Schaal argues that this relationship of heterogenous support of homogenous processes shows how respondent conditioning was, in a sense, its own concept.

**Operant Behavior**

Schnaitter (1999) discusses whether it is possible to reduce operant behavior to “physical events” (p.218). Actions, for Schnaitter, are "[...] the types of which biomechanical movement patterns (and in some cases their immediate physical consequences) are tokens" (p.218). The common effect of these movements individuates the class, and the number of
ways movements can do so is indefinite. The flipping of a switch may be accomplished by “moving the switch with the fingers, rubbing one’s back against it (as might be done if one’s hands are full), or even through bizarre movements such as lifting the switch with the tongue or standing on one’s hand flipping the switch with the toes.” (p.232). The instances are reducible to physical events, but not the class. The irreducibility of the operant class derives from different physical events being able realize the same functions.

**Aggregate Product**

In the context of metacontingencies, Houmanfar & Rodrigues (2006) argue for the irreducibility of aggregate product to interlocking behavioral contingencies. This irreducibility derives from possible variation in division of labor upon which the metacontingency is not directly contingent. They give the example of two people cooperating differently to produce the same meal. "The meals would not occur without their behaviors, however, the aggregate outcome of their interlocked behavior, the meal, is 'more than' or 'different than' the cumulative effect of their individual behaviors (p.21)". This is one of the reasons to believe, they think, that aggregate product is an “emergent” property. In a later article, Houmanfar and Rodrigues (2010) embark on an ontological narrative to justify aggregate product as such a property. They describe emergence as a "qualitative and substantive difference between the two levels" (p.83).

These examples indicate multiple realizability as a recurring theme in the context of structure-function relations, as well as level-hood through continuity and autonomy. This article carries this argument further, by suggesting that *structure-function relations as data is operationalized multiple realizability*. The next section sketches structure and function in this spirit.
Structure and Function as Multiple Realizability

The previous section suggested how multiple realizability matters. This section operationalizes multiple realizability. This is an argument for irreducibility. Its validity hinges upon, firstly, the definition of irreducibility, and secondly, the alleged reason of why multiple realizability implies said irreducibility. The different ways “reduction” can be defined are legion (e.g., Brigandt & Love, 2008; Sarkar, 2008; van Gulick, 2001; van Riel & Van Gulick, 2014). This article eschews such grand projects and focuses on a sparser claim of irreducibility: the functional relation in an experimental context. The idea was indicated in the discussion of substitutability. Functional relations are irreducible to the degree that different relata can realize those relations. When we define these relations for the purpose of experimental control, the different relata that can realize such relations are intrinsic to our concern. In other words, we do not understand the scope and limitations of the experimental control of a functional relation unless we understand its multiple realizability.

There are several conceptions of multiple realizability (e.g., Funkhouser, 2007; Gillett, 2003; Klein, 2008; Richardson, 2008; Shapiro, 2000). They are all influences for the account to follow, but note two differences. First, these authors ground the meaning of multiple realizability in metaphysics or phenomena. The current definition grounds the concept in experimental control. Second, all of them attempt to build significance into the definition itself. By contrast, the definition given here will be a data-based definition that requires supplementary assumptions to avoid triviality. The following section reiterates the listed components of structure as function – through the lens of multiple realizability – as three parts: (i) convergence, (ii) divergence, and (iii) relevance.

Convergence

(i) The convergence should capture the realization-part of multiple realizability. The realized properties of interest are functional relations. Three points will be made. First,
convergence criteria render multiple realizability empirical. Second, structural status is empirically relative to convergence criteria. Third, convergence criteria can be viewed as the empirical scale (or level of analysis) at which functional level-hood is charted.

If structure-function is understood as multiple realizability, any critique of the latter may transfer to the former. One critique is that multiple realizability is just idealization (Klein, 2008). Schnaitter’s (1999) version is vulnerable to this critique. Irreducibility is said to obtain because the list of structures that would entail a switch closure is indefinite. There is a problem. The sameness of biomechanical movement is determined solely by the switch closure itself. Imagine a random spread of black dots on a whiteboard and a circle drawn around some of them. The issue is what the dots within the circle have in common – other than the circle itself. The switch closure is similar. It is an idealized delineation of biomechanical movement, and that is the only reason the list of possible structures is indefinite. So, all that stands between the functional unit and reduction is stubborn insistence upon terminology (or where to draw the circle). To say that the idealization is pragmatic does not help. It only obscures the conceptual question: “What makes it useful?”

What is lacking is the causal relation; a relation that the various structures must converge on. We do not need indefiniteness of structure when we have a convergence criterion. To argue for irreducibility, all we need is arbitrariness for which structure that serves as relata. Hence, two different movements plus a causal relation will suffice for making Schnaitter’s switch closures functional in a way that goes beyond idealization. Schaal’s (2003) example satisfies the causal requirement. He specifies causal relations (the respondents) in the discussion of how the different mollusks obeyed the same respondent rules. Schaal also notes that there were minor discrepancies that were accounted for on the neurological level. This shows that the respondent relations were also idealized to an extent. However, we can now speak of degrees of idealization. The less idealization, the more
empirical content. Complete idealization is convergence by definition – that is, someone just drew a circle around the dots and called it multiple realization.

Convergence criteria are cherry-picked. We favor certain types of criteria and keep the number low. This illuminates the relative and empirical nature of structure-function relations. Take someone drinking a glass of water using his left or right hand. Assume a difference in the volume consumed given left-right variation. The right-hand drinking is modeled as \( f(x) = 2x \) and left-hand drinking as \( f(x) = 1.5x \) (with \( x \) representing deciliters). It turns out the data points do not overlap perfectly. If perfect overlap of data points was the convergence criterion, the left-right hand difference would be a confound for those interested in that degree of experimental control. However, on a margin of error of say 1-2 deciliters, the left and right hand do converge on volume. Also, if the convergence criterion was linearity then the hands converge perfectly. The margin of error is the degree of idealization. The linearity or proximity of data points are different convergence criteria. We could also plot the relation with the hands as the dependent variable. That is yet another convergence criterion. We can only ascertain structural status in the context of such criteria.

Functional classification also hinges on keeping the number of criteria low. It has been pointed out (in different terminology) that the more convergence criteria we add, the less likely are divergent structures (Ylikoski, 2013). For example, we may be inclined to say that ordinary and e-cigarettes are functionally interchangeable, but they are only so if we limit the claim to, say, nicotine delivery. They may not weigh the same, cost the same, or have the same health effects. Another example is digital and analog clocks, an example also used by Schnaitter (1999). Both have the salient property of displaying time, but that saliency distracts us from differences. Only the digital clock affords easy augmentations of timers and stopwatches. This is like comparing a wooden plank to a steel rod. The two can support an overlapping range of weights, but they obviously depart at some point along this and
additional dimensions. The importance of including failed convergences may be illustrated by an analogy. Consider how we might quantify discrimination, by dividing the discriminative stimuli rate by discriminative stimuli rate plus delta stimuli rate (Dinsmoor, 1951). Here, the quantitative meaning of discrimination partly hinges on delta stimuli. The quantitative meaning of functional level-hood may in a similar way hinge on failed convergences. Such consideration would be part of how one specifies the domain the function inhabits.

In sum, Klein (2008) and Ylikoski (2013) may have inadvertently given us empirical criteria to express functional level-hood. Degree of idealization and convergence criteria in tandem offers a scale at which structure-function relations can be expressed. Degree of idealization is the margin of error for recurrences of same process whilst convergence criteria are the dimensions along which those recurrences are examined. This might be helpful for developing a quantitative treatment of functional level-hood.

**Divergence**

(ii) If we are to speak of different structures, we need a measurement dimension to justify that difference. This will be referred to as a divergence criterion. This criterion captures the multiple-part of multiple realization. It will express from what we wish to claim irreducibility. When our measurement of divergence is trivial, one could argue that the functional irreducibility observed is also trivial. We may, for example, point to minuscule differences in biomechanical movements to explain functional classification. Depending, however, on what beliefs – that is, the background theories and empirical findings – we have about biomechanical movement we may marvel at such differences or dismiss them as unimpressive.

Richardson (2008) has (in different terminology) pointed out that any divergence criterion will not suffice. He might have approved of Schaal’s (2003) case study because Schaal references divergence validated as interesting by neurology. In other words, the
divergence was non-trivial because another science was invested in that difference – that is, the difference was inseparable from the validity of the neurological explanation. In that sense, it was a genuine difference and not a measurement artifact. Similarly, if we wish to say that operants are structural to metacontingencies, it is not enough to say that metacontingencies can have their individuals substituted (cf. Glenn, 2003). One must show that there is operant divergence rather than divergence of individuals. For example, the same aggregate product can be maintained by a receiving system and yet be internally maintained by negative or positive reinforcement.

Lastly, previous examples – analog or digital clocks, left- or right hand, wooden plank or steel rod, e-cigarette or cigarette, this movement or that movement – are all cases of nominal divergence. That is, the identified difference is expressed as a category variable, as opposed to ratio variables and the like. It is perhaps too easy to maintain structure-function distinctions with nominal divergence – just identify any minuscule difference and name it. Convergence will trivially follow. Nominal examples may also falsely dispose us to think of function as qualitatively distinct from structure. However, the distinction can be measured as a more fluid relationship. The water drinking example could have employed grip strength as the divergence criterion. Or, suppose we maintain the synchronized movement of two hands with reinforcement, with synchronicity as the convergence criterion, then the divergence criterion might be the hands’ absolute speed.

**Relevance**

(iii) Relevance is the linkage between divergence and convergence (Gillett, 2003). It demarcates topography from structure. That demarcation need not be flawlessly sharp. It need only steer us in the direction of what is constitutive of the function. What follows is a suggestion of how to do this. It relies on the behavior-analytic distinction between procedure and process (Catania, 2013) and some rudimentary set-theoretical thinking. This will move
the idea of continuity away from a spatial (part-whole or micro-macro) conception and towards a conception of continuity through operational definition.

This approach eschews structure as an empirical account of how parts, material, or architecture ‘make’ the function. Instead, we can view procedural structure as a mathematical derivative of our measurement conventions. That is, relevant features of functional variables should follow from the operational definition of that variable. The difference between topographical and structural variables was foreshadowed in the discussion of continuity. Topographical variables may be defined as those behavioral variables that can vary and be set to zero without negating a functional relation. Likewise, structural variables can also vary without negating a functional relation, but they will negate the relation if set to zero. For example, when a researcher measures instances of lever pressing, the rat could depress the lever with a variety of force. Procedural structure would here be force, because it can vary but not be set to zero. If structural variables constitute a range then it is the range that cannot be set to zero. For example, the value of the variables of pressing with left and right paw cannot both be set to zero without also negating the lever press. Notice that what was structure here followed from how the measurement of the functional variable was defined. This formal aspect has two implications. First, procedural structure has no empirical content. It simply follows from our measurement conventions that the structural variables cannot be set to zero. Second, we can exclude bizarre cases when relevance is understood formally. For example, the rat’s lever pressing would not occur if all oxygen was pumped out of the operant chamber. The oxygen deprivation will negate the behavior, but it will so do for empirical reasons and not definitional reasons.

Convergence and divergence criteria add the empirical dimensions. The question is which of the structures \( s_1 \ldots s_n \) that can serve as relata in the same functional relation, and by what measurement the structures \( s_1 \ldots s_n \) are different. Only some of these different structures
will adequately serve as relata. That residue is processual structure. This endows structure
with empirical content through functional relations. This is quite different from an interaction
type of structure and function. Structure and function were joined by definition. This is akin
to rejecting dualism by starting with a monistic language. In sum, we can put structure and
function together as convergence, divergence, and relevance:

(i) The structural variables $s_1...s_n$ converge on any relational aspect $C_1...C_n$ that is replicated
for the independent or dependent variable $v$;

(ii) all structural variables $s_1...s_n$ diverge from each other by criteria $D_1...D_n$; and,

(iii) the structural variables $s_1...s_n$ cannot be set to zero without negating variable $v$.

Suppose we operationalize “pets” as any non-human creature deliberately cared for by
humans in domestic settings. This measurement convention approves cats, dogs, fish… as
instantiations of pets. This allows us to conceptualize these animals as variables that are
algebraically substitutable for the pet variable. The pet variable has multiple solutions, which
follows from letting nominally divergent observations to qualify as pets. Dog, cat, fish…
‘solves’ for pet. Think of these solutions as a set with its elements representing >1 solution
for a functional variable $v$. This can be the dependent or independent variable. Everything so
far is the procedural dimension. We add the processual dimension by testing if this set has
any elements that can serve as the same relata. Hence, we need to pick a relation on which the
multiple solutions must converge. So, posit a functional relation between pet ownership and
stress relief. Some of what we have decided to accept as pets might be confounds relative to
this relation, and others might not be. The confounds, say fish, is not multiple realization; the
non-confounds, such as cats and dogs, are multiple realizations because they can serve as
relata.
Critique and Avenues of Research

Arbitrary substitutability

The critic might argue that substitutability is never arbitrary. Strictly speaking, this is correct. Organisms are not automatic fail-safe systems that switch between alternative structures in a graceful and context free fashion. Three brief points will be made on this issue. First, an organism’s performance may be perturbed if structural variables are altered. The response rate of a rat might be disrupted if we stopped reinforcing or block some its biomechanical options for lever pressing. People might recover from brain damage but it takes time to do so. However, arbitrariness obtains to the extent this disruption is transient and negligible in magnitude. To that extent, we can abstract away such effects like friction is occasionally abstracted away in physics (Ladyman, 2014). Second, arbitrariness can be defined for different scales. It may apply across consecutive instances of behavior, aggregate performance, experimental conditions, experimental sessions, groups, or populations. These are quite different cases. It may for example be arbitrary how group members interact to solve a task, but not arbitrary that the groups must solve that task in a specific way. That is, the arbitrariness across groups but not within groups. Third, arbitrariness might be viewed as a dispositional property applying to any of these scales. From this perspective, arbitrariness is not endogenous ability but exogenous if-then potential.

Heuristics

Two points on heuristics will be made. First, structure-function relations should not be viewed as the search for negative results. It may appear so since the checking for convergence is a non-effect report on variation in structure. A different way to carry out such investigation is to look for sources and boundaries of multiple realization. One avenue of positive research is whether Rosenberg’s claim can be supported; namely, that selection produce structure-function relations. Another avenue of positive research is to use of the dimensions of
idealization and convergence criteria to map multiple realization onto accepted function units; that is, to discover their boundaries. Second, this definition of structure-function relations was made broad and permissive so that other researchers can freely add restrictions as they see fit. Also, these basic terms can be used to investigate structure-function relations more systematically. We can inquire what convergence, divergence, and relevance criteria that were used, and why.

Non-trivial structure-function relations

This sparse and permissive definition is for those wishing to take a step back from grand emergence theorizing (cf. Bedau & Humphreys, 2008) and ask: what must the data look like? The advantage of a data-based definition is broad applicability. Also, it is easier to provide a relevance criterion for variables than for phenomena. The latter may differ between cases and require empirical knowledge not yet available. The disadvantage is the lack of a straight path between such results and an anti-reductionist argument for the researcher’s preferred functional entities, such as operants or culturants (Glenn et al., 2016). The definition offered in this article leaves it to the experimenter’s good judgement (Steinle, 2002) to select interesting variables, or the experimental design and supplementary theoretical assumptions to select variables that saves the phenomenon of interest (Bogen & Woodward, 1988).

Related research

There are two fields of research in behavior analysis that to some degree relate to structure-function relations. First, there is variability research. (e.g., Neuringer, 2002). Research into structure-function relations is not the same as research into variability. There is no convergence criterion in variability research. In other words, variability research aim to understand the sources of behavioral divergence, and not divergence and convergence in tandem. In variability research one may ask under what circumstances a pigeon would vary its pecking among alternative keys. In the investigation of structure-function relations, the
question might be which of these pecking variations that can act as the same relata in some other behavioral process. For example, will all these variations in pecking act uniformly when targeted by a different schedule, or does some of these variations predict subsequent behavior? As argued, structure is relative to the relations picked. However, variation is a necessary part of testing whether an aspect of behavior is structural relative to some other functional relation. Hence, knowledge of the sources of variability can certainly help in this investigation.

Second, there is stimulus equivalence research (e.g., Sidman, 1992). There is a similar interest here with functional interchangeability. The main difference is that functional identity is limited to the set-theoretical definition in equivalence research. Here, functional identity simply mean relationality, and so has no constraints on the kinds of relations needed for variables to be identified as structural. Perhaps constraints are needed, but that is beyond the scope of this paper. That said, both are related since they are concerned with functional interchangeability through structurally divergent variables.

Conclusion

This article proposed that structure-function relation may be understood as functional level-hood, and that structure-function relations as data is operationalized multiple realizability. On this view, functional level-hood is the empirical fact when divergent properties are free to arbitrarily serve as relata for same relation. The convergence criterion defines the higher-level and the divergence criterion defines the lower-level. Functional level-hood is not a spatial concept. The higher-lower level distinction is an epistemological difference and a relevance criterion links structural variables to functional variables by operational definition. This is a move towards an observable and quantitative conception of structure-function relations.
References


Experimental section

Dynamic Agent-Environment Fit in Ecological Psychology
Abstract

This study has two aims. First, to introduce dimensionless ratios and the concept of dynamic agent-environment fit from ecological psychology. This is exemplified in the context of time-based avoidance behavior in a ball transportation task. Plastic play-pen balls were delivered 8 meters away from a wooden box on a non-contingent interval schedule. Participants were asked to transport the delivered balls into the box. The balls had to be transported rapidly, so that new balls would not stack at the delivery location. The delivery intervals would change gradually from 2-12 or 12-2 seconds. Participants learned to adapt to these changing conditions and time their transportation attempts to avoid stacking. The temporal properties of this avoidance behavior, when considered in proportion to the delivery interval, proved to be an invariant ratio regardless of absolute time. Participants spent 80% of the delivery interval to move towards the box and back. In addition, they moved faster towards the box than back to the delivery location. The second aim is to exemplify how structure-function relations can be interpreted as data. The observed scale invariance is used as a reference point to do so. The participants had different preferences for how close, and when, to move towards the box. These differences entailed that participants instantiated their relative timing at different velocities. The diverse velocities converged on often equally accurate approximations of the scale invariant ratio. It is suggested that velocity is a structural dimension of the time-based avoidance behavior. The paper concludes by discussing the benefits of interdisciplinary work between behavior analysis and ecological psychology, and the empirical study of structure-function relations.

Keywords: interval schedules, avoidance, structure, function, ecological psychology, behavior analysis, dimensionless ratios.
Behavior analysis and ecological psychology have certain striking similarities and perhaps complementary differences. The relationship between the two disciplines has been discussed in broad historical and theoretical detail by Morris (2003, 2009), and this seems to be the only extensive treatment of the topic. Both disciplines approach behavior from a perspective that might be termed functional, relational, or contextual. Behavior analysts classify behavior in terms of the whole organism’s interactions with contingencies in the environment. Ecological psychologists classify perception in relation to action in the environment. Both disciplines endeavor to show that dualistic explanations of behavior are redundant. Behavior analysts deny that a mind initiates action and ecological psychologists deny that representations mediate perception. The behavior analysts appeal to the selection of behavior through contingent relations between actions and consequences. The self-organizing process of selection is sufficient to explain the origin of action without mental states (Skinner, 1981). The ecological psychologists appeal to information available to the organism in the environment as sufficient for guiding action. That information reveals itself as sufficient when perception is considered relative to action, and thereby the need for intermediate representations dissolves (e.g., Chemero, 2009; Fajen, 2007; Warren, 1984).

Morris (2009) concludes his review of Harry Heft’s book “Ecological psychology in Context” by calling for integrative work between behavior analysis and ecological psychology. This part of the thesis heeds Morris’ call, and does so in the context of the behavior-analytic distinction between structure and function. A theoretical interpretation of the structure-function distinction within behavior analysis was outlined in Viklund (2017). This provides a basis for discussing a specific ecological concept – soon to be introduced –

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2 This article was produced in cooperation with the University of Cincinnati at the Center for Cognition, Action & Perception, and the Oslo and Akershus University College of Applied Science at the Department of Behavioral Science.
and a related experimental finding. In other words, the experiment described here examines structure-function relations in an interdisciplinary setting, and so continues what Morris started, and also includes an investigation of structure-function relations. In this context, the value of integration is demonstrated if an ecological approach helps to enrich the behavior-analytic distinction between structure and function.

The experiment to follow concerns the ecological notion of dynamic agent-environment fit. A classic example is found in a study by Warren (1984). It brought James Gibson’s ecological theory of perception into an experimental setting (Gibson, 1979). Warren derived two hypotheses from Gibson. First, organisms perceive the environment in terms of action possibilities – that is, in terms of what organisms can do. Gibson chose to call these possibilities “affordances”. Warren described affordances as “the functional utility of an object for an animal [agent] with certain action capabilities” (p.683). Second, affordances are organism-environment relative properties. Measurements that pay homage to this idea are “intrinsic” in ecological terminology. Intrinsic measurements are dimensions of the environment relativized to an organism-action system. This is contrasted with extrinsic measurements. These express the relationship between the organism and the environment in terms of independent or context-free properties, for example absolute time and distance. Warren considered extrinsic measurements to be arbitrary, in the sense that the organism and the environmental dimensions are not jointly determinative of the chosen unit. Intrinsic measurements may capture the dynamic agent-environment fit of an organism, defined by Warren as a “specific set of values of the animal [agent] and environmental system properties that are relevant to a given activity” (p. 684). The concept of affordances and the premise of direct perception are central ideas in ecological psychology (Chemero, 2009). Killeen and Jacobs (2016) offer a recent discussion of what affordances can add to behavior-analytic
theory. This paper focuses solely on Warren’s approach to dynamic agent-environment fit, and can be illustrated in his famous stair climbing experiment.

Warren (1984) investigated what aspects of stairways that participants perceive and how these perceptions are related to possibilities of action in stair climbing. The examined variable was the ratio of participant leg length over stair riser height. This was the intrinsic measurement, not only because it was an organism-environment relation, but also because leg length and riser height were of the same dimensions. Since the ratio was one of length to length, it was a dimensionless ratio and so yielded a pure number – it named just relative length rather than the relationship between two heterogenous units. Warren referred to dimensionless ratios as pi-numbers. He hypothesized that pi-numbers – as opposed to extrinsic metrics – would capture and predict the occurring perception-action relations in stair climbing. To demonstrate this, participants of different leg lengths were recruited and sorted into a tall and short group. The first experiment investigated how participants estimated whether they could climb stairs in bipedal fashion. Slides of stairs with different riser heights were presented and participants were asked to rate how confident they were of the stairs’ climb-ability. Using a biomechanical model, Warren predicted that participants would estimate the stairs as not climbable if the ratio of leg length over riser height became 0.88. At this point, participants would no longer be able to place their foot flat on the next step. Hence, it would be the critical point at which participants would need to switch to quadrupedal hands-and-knees climbing. The 0.88 ratio confirmed the preferences of both groups and so demonstrated how the ratio applied across different extrinsic values. An optimal ratio was obtained in a second experiment. Participants were asked to walk on a motor-driven staircase and their oxygen consumption was measured. The ideal length to riser height for least energy consumption was estimated at 0.25. The acquired optimal ratio predicted the preferences of both the tall and short group in a third experiment. Participants were shown slides of
staircases and asked to choose between stairs of different riser heights, based on which option that seemed more comfortable. Their verbal preferences correlated with the optimal ratio of 0.25 for both the tall and short group.

Warren’s experiments showed how perception connected to action possibilities and how the pi-numbers – that is, the critical and optimal ratios – proved constant through differences of scale in the tall and short participants. The dynamic agent-environment fit was scale invariant. The findings were summarized as follows. “The effects of adopting natural or intrinsic units of measurement are underscored by the near congruence of group curves when plotted on intrinsic axes, thereby “annihilating” group differences and rendering them functionally equivalent” (Warren, 1984, p. 700). The study demonstrated how a relational approach to behavior could unify seemingly different results under the same higher-order variable.

Warren’s (1984) study has inspired generations of ecological psychologists (Chemero, 2009). A review of the research and theory is found in Fajen, Riley, and Turvey (2009). These authors make a distinction relevant here, between body-scaled affordances and action-scaled affordances. Both involve opportunities for behavior, but in different ways. Body-scaled affordances are possibilities for action constrained by environment-to-body relations, such as leg over riser height in Warren (1984). Action-scaled affordances are constrained by environment-to-behavior relations. For instance, the affordance of intercepting a tossed ball requires a relationship between ball velocity and running velocity.

The aim here is twofold. First, to exemplify how pi-numbers (i.e. dimensionless ratios) can capture avoidance behavior on progressive and non-contingent interval schedules. This is thereby an investigation into action-scaled behavior, where temporal dimensions of behavior are related to temporal dimensions of an interval schedule. This will be successful if said behavior can be described in terms of scale invariant pi-numbers, and thus capture a
dynamic agent-environment fit for time-based avoidance. If successful, it would be an example of how pi-numbers – an idea from ecological psychology – can be useful in the investigation of schedule performance.

The second aim is to show how the obtained dynamic agent-environment fit has structural and functional dimensions. This part will employ the ideas sketched in Viklund (2017) of how to approach structure-function relations as an empirical concept. The fundamental idea was that structure and function relationships obtain when divergent relata (structure) converge on the same relation (function). The hope is that this will bring further clarity to the structure-function distinction and offer an additional angle for interdisciplinary thinking.

These aims were originally intended to be one and the same. They were split for theoretical reasons (see discussion). Two connections remain. First, it was argued that structure is a relative concept. Structural status is relative to functional relations (Viklund, 2017). The scale invariant pi-numbers will serve as the reference for evaluating the structure-function relations in this experiment. Reliable functional relations are the standard for investigating structure-function relations. Such a relation is provided if the first aim is achieved. Should one wish to separate these aims, a different functional relation would have to be provided. Second, and more generally, the common denominator of both aims is their concern with the meaning and advancement of relational or contextual approaches to behavior (Morris, 1988). When these two aims are phrased as research questions they read as follows. (1) Can pi-numbers capture a dynamic agent-environment fit for time-based avoidance? (2) What possible structure-function relations might such a fit have? This is investigated in a ball transportation task.
Method

Subjects

Twelve participants were recruited for course credit at the University of Cincinnati. Two were excluded due to neglecting the rules of the procedure. Of the remaining ten, five were males and five were females, and age ranged between 18 and 23. Participants are numbered in the order they partook in the experiment.

Setting

The experiment took place in a room with a wooden box situated in one corner. The box was 110 cm broad, 50 cm deep, and 120 cm high. Behind the box, and on each side, were black curtains. A white plastic tube was situated in the diagonally opposite corner of the room. Half the tube was obscured by a black opaque curtain. The experimenter sat behind this curtain with a box of plastic play-pen balls. The tube was tilted at an angle with the bottom side resting on a small cardboard platform at waist height. The diagonal distance between the tube and the box was 8 meters. White tape on the floor marked the distance from the box to the tube. The tape allowed the experimenter to assess a participant’s distance from the box and the tube with a precision of 25 centimeters. A GoPro Hero 1 camera was attached in the ceiling so that the delivery platform, the tape marked distance, and the wooden box came into view. See figure 1 for an illustration.

Procedure

The plastic play-pen balls were delivered manually through the tube at specific intervals, henceforth referred to as delivery intervals. This was the independent variable. The experimenter would look at a computer screen and rely on a program in MATLAB to specify when to deliver a ball. A ball would be inserted into the tube, roll out on the cardboard platform, and come into view for the participant.

After signing a consent form, the GoPro camera was activated. All participants were told that the goal was to transport balls delivered through the tube into the wooden box as
accurately as possible. Participants were only allowed to grab one ball at the time. Each ball could only be used once, so missed or dropped balls were to be ignored. It was emphasized that participants were free to transport the balls however they liked. Any combination of throwing, speed, and distance was allowed. All participants received a training session with these rules. An additional rule was added for the experimental conditions. Participants were told to say “stop” if two balls stacked at the delivery platform. If two balls stacked, the schedule would be restarted as many times as necessary for the participant to complete the schedule without stacking. The intent was to negatively reinforce ball transportation behavior that was compatible with non-stacking, and to define stable avoidance behavior as free of stacking incidents.

The participants were divided into two groups, denoted as AB and BA. The training condition of Group AB consisted of a non-contingent progressive schedule that delivered balls at certain intervals. The ball delivery intervals progressed from 12 seconds to 2 seconds. The delivery interval increased or decreased with one 1 second for each consecutive ball delivery. After repeating the 2 second interval once, the schedule would progress back to 12 seconds in the same manner. So, the parametric change in the delivery intervals for group AB were slow-to-fast and fast-to-slow. Group BA received the same training, except the intervals would start at 2 seconds and progress to 12 and then back to 2 seconds. So, the order of exposition for parametric change was reversed for BA, being fast-to-slow and slow-to-fast. The experimental conditions were the same as the training session except for three differences. First, the non-stacking rule was imposed. Second, the 12-2 (condition A) and 2-12 (condition B) schedules was separated by a break. The experimenter would empty the wooden box and verbally congratulate the participant on completing the series without stacking, saying for example “Great! You made it.” Third, the number of balls for each delivery interval was increased to 4 balls. Hence, 44 balls were delivered for each condition.
Table 1 provides an overview of the procedure and experimental conditions. As this was a parametric design, there was no independent baseline. The AB and BA group completed a four-squared matrix to control for direction of parametric change and history effects. This experiment is analyzed in the single-subject design research tradition and so the results are assessed and presented for visual inspection (Kazdin, 2011; Sidman, 1960).

**Data gathering**

The data was manually coded by analyzing the GoPro recordings in Windows Media Player Classic. The time display in this program rounds to the nearest second so the grain of this data analysis was in seconds, with a give or take 500 milliseconds loss of precision. Time stamps were made when participants grabbed a new ball, when a ball was released, and when the participant returned to the delivery platform. Grab time was coded when the participant’s hands enveloped a new ball. Release time was coded when the participant let go of the ball. Return time was coded after the ball had been released and when the participant placed one foot flat within 1 meter of the delivery platform. Upon releasing the ball, participants would come to a halt at a specific distance from the box. The experimenter would note the position of the participant’s front foot during such halts, and rely on the tape markings to code for distance. This produced a vast amount of data. A third-party agreed to re-code the first participant of each group to sample an inter-observer agreement. Total duration IOA, and mean duration IOA, were calculated separately for the time stamp series of grab, release, return, and distance (treated as duration). Both measurements, for all four units, yielded an agreement above 95%.

**Data analysis**

Three intervals were derived from the grab, release, and return time stamps. The grab-to-return interval was defined as the movement interval. This was the time taken between grabbing a ball and placing one foot flat within one meter of the delivery platform (after
having released a ball). The throw interval was the grab-to-release time, or time taken between grabbing the ball and throwing it. The return interval was the release-to-return time, or time taken to release the ball and return to the delivery platform. The movement interval is the sum of the throw and return interval. In addition to the intervals, the number of meters that participants would locomote from the delivery platform towards the box are referred to as distance. Distance was only measured when participants ventured beyond one meter of the delivery platform. This excluded ambiguous cases where the participant would just shift around while throwing the ball without any running or walking.

Two pi-numbers and two measurements of velocity were derived. The first pi-number was the ratio of the movement interval over the delivery interval, and is abbreviated as $\Pi_{\text{movement}}$. That is, $\Pi_{\text{movement}} = \frac{\text{movement interval}}{\text{delivery interval}}$. The second pi-number was derived from within the movement interval. This was the ratio of the throw interval over the return interval, and is abbreviated as $\Pi_{\text{throw}}$. That is, $\Pi_{\text{throw}} = \frac{\text{throw interval}}{\text{return interval}}$. Two types of velocities were quantified. Actual velocity $= \frac{\text{distance} \cdot 2}{\text{movement interval}}$. Distance was multiplied by 2 to include distance when returning to the delivery platform. The last derived measurement was ideal velocity. This was used to analyze the relationship between the pi-numbers and distance. If participants sought an invariant value of $\Pi_{\text{movement}}$, then any distance they chose to move across would entail a velocity expressed as follows. Ideal velocity $= \frac{\text{distance} \cdot 2}{\text{delivery interval} \cdot \Pi_{\text{movement}}}$. Recall that $\Pi_{\text{movement}}$ is a pure number and presumed to be invariant in this equation. A match between ideal and actual velocity indicates approximation of $\Pi_{\text{movement}}$, and difference across equally accurate matches clarifies that different velocities were employed to satisfy the same requirement (see results and discussion section). Table 2 provides an overview of all dependent variables and coding conventions.
The excluded data were as follows. The delivery intervals of 2 and 3 seconds had to be cut from all analysis of $\Pi_{\text{throw}}$ and velocity. The participant would rarely venture beyond 1 meter of the delivery platform on these intervals, and so no unambiguous measurement of velocity could be obtained. In addition, the throw and return intervals would frequently be logged as zero for these intervals, making it impossible to extract a ratio. The return intervals were only logged as zero on two trials when the delivery interval was at 4 seconds or above. Performance on these trials were excluded. Lastly, all graphs only display performance for the conditions that participants completed without balls stacking. Table 3 provides an overview of re-runs on each condition and session time.

**Results**

**Pi-movement**

The $\Pi_{\text{movement}}$ ratio appeared to be scale invariant. Figure 2 displays individual performance for $\Pi_{\text{movement}}$ in relation to parametric change and conditions. Visual analysis suggests that $\Pi_{\text{movement}}$ approximates 0.8. That is, participants would spend 80% of their allotted time moving towards the box and back, and 20% standing still waiting for a new ball. Figure 2 suggests that this scale invariance replicates through the direction of parametric change and order of exposition, suggesting good experimental control. Adjustments appear more frequent in the first condition for both AB and BA, as indicated by the flatter curves. An exception is participant 10. This is perhaps due to being the only participant that neglected the rule of not reusing balls. Figure 3 summarizes the same performance as AB and BA group averages.

**Pi-throw**

The value of $\Pi_{\text{throw}}$ reliably stayed < 1 and often approximated 0.8 or less. Participant 1, 3, 9, and 7 kept a ratio slightly above 0.8. Figure 4 displays this finding as individual performance for $\Pi_{\text{throw}}$. These ratios entail that participants preferred to move faster towards
the box than back to the delivery platform. The preference appears invariant through parametric change and order of exposition, although the data is somewhat more variable than pi-movement. If the throw and return intervals were instead expressed as ratios over the delivery interval, their values would approximate 0.35 and 0.45 respectively (see discussion).

Figure 5 provides an overview of \( \Pi_{\text{throw}} \) as AB and BA group averages, and suggests 0.8 as the central tendency. Standard deviation error bars indicate notable departure from this mean.

**Distance moved**

The data for distance moved was less orderly and hence only individual data is presented. Figure 6 displays individual data for absolute distance moved. The delivery intervals did not seem to control distance moved in the same linear fashion. Overall, the participants moved closer to the box when given more time, but the transition points from longer or shorter distances varied individually. It was gradual on some occasions and at other times seemingly non-linear. For instance, participant 9 exemplifies a smooth transition, participant 7 is sharper, and participant 8 was remarkably stable.

**Velocity**

The variation in distance moved implies that participants maintained the same timekeeping at different velocities. Actual velocity (= distance moved \( \cdot \) 2) \( \div \) movement interval) was compared to ideal velocity = (distance moved \( \cdot \) 2) \( \div \) (delivery interval \( \cdot \) 0.8) to clarify this relationship. The average distance moved in four trials (i.e. performance per delivery interval) was used to calculate ideal velocity for all participants. This was compared to the participants’ actual velocity, being their average distance moved for every four trials over their average movement interval for the same trials. This clarifies, in two respects, how participants used different velocities in their pursuit of \( \Pi_{\text{movement}} \) at sometimes equal level of accuracy.
First, the match between ideal and actual velocity can happen at different values within individual performance. Figure 7 displays the maximum and minimum actual velocity that each participant attained, and for which the difference to ideal velocity was less than 10%. Figure 7 shows how velocities that approximated Π_{movement} could vary within individual performance. Second, figure 7 also suggests that approximation could vary across participants. Figure 8 clarifies this relationship by including all performance as total averages of ideal and actual velocity for each participant, and demonstrates how velocities could be different across individuals and yet match well with ideal velocity.

**Discussion**

The first research question was whether pi-numbers can capture a dynamic agent-environment fit for time-based avoidance. The results suggest that Π_{movement} and Π_{throw} can describe such a fit. These pi-numbers describe the temporal dimensions of the present avoidance behavior as invariant through differences of parametric change and order of exposition. The dynamic is one of continuously allocating the same proportion of movement time as a safety margin, and continuously oscillating between a higher velocity when moving towards the box than when moving back to the delivery platform. Since both pi-numbers approximated 0.8, the dynamic agent-environment fit can be formally idealized as Π_{movement} = Π_{throw}.

One shortcoming of this experiment was that the pi-numbers were not manipulated. This was an essential aspect of Warren’s study (1984). In this case, no insight could be gained concerning the functional level at which the two pi-numbers appeared to correlate. However, Warren required three separate experiments for such manipulation. A similar path might be taken here. If so, one methodological challenge is how to manipulate Π_{movement} and Π_{throw}. These action-scaled behaviors are more abstract than Warren’s length to riser height ratios.
Perhaps a different schedule of negative reinforcement would produce pi-numbers at difference values, or the pi-numbers might be directly targeted by reinforcement.

Another open question, of concern to both ecological psychology and behavior analysis, is how participants modulated and maintained their relative timing. This experiment is informative of the dynamics of time-based avoidance preferences and not temporal perception. From a behavior-analytic standpoint, the results and design can be placed closer to the research tradition of choice (Poling, Edwards, Weeden, & Foster, 2011) than timing (Lejeune, Richelle, & Wearden, 2006). Participants were free to choose any safety margin and the strategy to instantiate it. They invariantly chose a specific proportion of the delivery interval. An open question is if this finding can be connected to matching laws studies. A striking similarity is that both involve scale invariant preferences. The strict matching law can also be stated as the relationship between two pi-numbers, being a response-to-response ratio and a reinforcer-to-reinforcer ratio. The indicated matching here is however a case of self-similarity and not a distribution between multiple operandia.

Knowledge of time-based avoidance preferences may also be useful in temporal perception research, as it provides such research with specific targets to be explained. The results obtained suggest that the object of perception may not be absolute time, just as the object of perception in stair climbing was not absolute length and riser height. Absolute speed and distance did not appear to specify that information, or serve as mediating behavior. These varied notably across participants. Had they served a role in specifying the necessary information, their variation should have predicted differences in timing (e.g., Laties, Weiss, & Weiss, 1969).

The data for the throw and return interval was expressed as $\Pi_{\text{throw}}$, as opposed to ratios in relation to the delivery interval, to emphasize self-similarity. The throw interval divided by the return interval appeared as a smaller copy of the movement interval divided by the
delivery interval. Three things can be noted. First, this is potentially an aspect of how participants keep time – they keep one safety margin within another. If one safety margin is compromised, the other can act as a buffer. If participants run faster towards the box, they should be able to match that speed on the way back if necessary. Not having to do so may provide some information of whether they are timing their behavior correctly. Unfortunately, the current data did not allow for this relationship to be directly confirmed; the one second grain of analysis had its limits. A more precise recording method would be recommended in future studies. More trials on each delivery interval may also help to obtain more stable behavior. Second, self-similarity is itself a potential research avenue in timing or avoidance generally. Is self-similarity for safety margins across different scales common? Or more generally, does avoidance behavior exhibit self-similarity on different scales? Third, self-similarity was part of the idealized interpretation of the dynamic agent-environment fit as 

\[ \Pi_{\text{movement}} = \Pi_{\text{throw}}. \]

The results exemplify how pi-numbers can be useful in interpreting schedule performance. A more general avenue for future research is to investigate schedule performance more in terms of dimensionless ratios. Temporal reinforcement schedules have traditionally been plotted against extrinsic units, such as interval to response frequency (Catania, 2013). In some cases, this may hinder a more general understanding of schedule performance. Pi-numbers can reveal seemingly disparate data as the same phenomenon asserting itself on different scales.

The second research question was if any structure-function relations could be demonstrated for this dynamic agent-environment fit. It was suggested in Viklund (2017) that structure-function relations can be interpreted as a sparse type of multiple realizability. The functional relation is the process to be realized, and the structures are the multitude that realize it. A functional relation is instantiated by things, or relata, that enter into a specific
relationship. Different relata can take on the same functional role by virtue of entering into the same relationship. To the extent different relata can enter into the same relation, that relation is multiply realizable. Any divergent properties of the relata, that are relevant for realizing a functional relation, and yet can vary, counts as structure. On this view, structure and function are not independent concepts but complimentary pairs. To count as structure and function, variables need to enter into a three-part constellation. First, the variables must convergence on the same relation. Second, the variables must diverge or be disparate from each other. Third, the divergent aspect must be a relevant part of instantiating said relation.

This take on structure-function relations yields two candidates in this experiment. These were (a) the absolute time scale at which participants operated, and (b) the velocity used to instantiate their relative timing. This experiment was carried out on the assumption that the first candidate (a) was the correct one. That is, scale invariance was a type of multiple realizability. It seemed to satisfy the criteria of structure-function relations. Divergent absolute time converged on the same pi-numbers, and absolute time was relevant for instantiating the pi-numbers. Hence, absolute time would qualify as structure and the pi-number invariance as function. The problem is that absolute time is also part of the stated relationship. In other words, the degree of scale invariance for $\Pi_{\text{movement}}$ is the context in which the relationship holds. If absolute time is structure, then structure reduces to the familiar concept of generality. This makes the concept of structure redundant. Therefore, the notion that absolute time counts as structure is here considered theoretically indefensible. For this reason, it was necessary to present this study as dual purposed, rather than as having a fated connection to structure-function relations.

Velocity is a more plausible candidate for structural status. The relationship velocity to $\Pi_{\text{movement}}$ reduces to neither generality nor variability. Convergence upon the $\Pi_{\text{movement}}$ invariance was quantified as a match between ideal and actual velocity. Figure 7 and 8 show
that this matching occurred and that it happened at divergent velocities. Also, velocity was relevant for instantiating $\Pi_{\text{movement}}$ given that participants moved across the room. This is still a description of generality in that $\Pi_{\text{movement}}$ was invariant through differences of velocity; but unlike absolute time, velocity was not itself part of the relationship described. Rather, velocity was part of the behavior that instantiated that relationship. One problem with conceptualizing velocity as structure is that $\Pi_{\text{movement}}$ could be measured while participants were stationary. This was due to the convention of always coding for return time after release time. This meant that velocity was not relevant for all instances of $\Pi_{\text{movement}}$.

The relevance criterion advocated in Viklund (2017) was that structure and function should be linked by definition. The function should, by definition, not instantiate if all structural variables are set to zero. Problems arise when this approach is brought to bear on absolute time or velocity. When applied to absolute time, structure reduces to the generality of the function; and velocity does not completely qualify because participants could instantiate $\Pi_{\text{movement}}$ while stationary. Velocity could only be assessed as relevant after the fact, and only for a subset of $\Pi_{\text{movement}}$. This shows that the conceptual issues of structure-function relations as data have not been fully resolved. This experiment was more successful in demonstrating a divergence-convergence relation. Velocity was the divergence and the pi-number invariance was the convergence. What is needed is a perspicuous account of what distinguishes variables that are constitutive of the function (structure) from variables that merely accompany it (topography).

Should any future research continue in this vein, a central question is what narrows and increases the range of possible structures, and whether such findings are not reducible to already known principles. The primary difference between this question and variability is that structural variation must converge on some other empirical process. In this case, different velocities converged on the $\Pi_{\text{movement}}$ invariance. The current results allow a comment on the
methodology of such investigations. If structure is relative to function, it follows that structural status can only be assessed to the extent there is good experimental control of the function. The scale invariant pi-numbers proved to be solid ground for such assessment, due to their remarkable stability. The calculation of ideal velocity relied on this stability. Future empirical claims about structure would require a similar level of control.

**Conclusion**

The present experiment shows how pi-numbers can be useful in the interpretation of time-based avoidance. The dynamics of time-based avoidance can be idealized to a match between $\Pi_{\text{movement}}$ and $\Pi_{\text{throw}}$. The same findings allowed for an attempt to demonstrate structure-function relations as data, and the pi-numbers provided an empirically stable reference point in this assessment. Velocity is a plausible candidate, but require additional theoretical justification to fully qualify. The distinction between what is constitutive of the function and that which merely accompanies it need to be clarified. However, velocity converged on $\Pi_{\text{movement}}$ at divergent values, and was instrumental for instantiating $\Pi_{\text{movement}}$ at any occasion that the participant moved across the room. If accepted as a structure-function relation, this experiment shows how such relations are not necessarily tied to specific functional concepts, such as operants. The behavior-analytic distinction between structure and function can be applied to other relational classifications, such as dynamic agent-environment fit in ecological psychology.
References


Killeen, P. R., & Jacobs, K. W. (2016). Coal is not black, snow is not white, food is not a reinforcer: The roles of affordances and dispositions in the analysis of behavior. *The Behavior analyst, 1*-22. doi: 10.1007/s40614-016-0080-7


### Table 1

*Displays procedure in chronological steps and events involved.*

<table>
<thead>
<tr>
<th>Steps</th>
<th>Events</th>
</tr>
</thead>
<tbody>
<tr>
<td>Received instructions</td>
<td>“The task is to get the balls into the box. How fast you move, or how close to the box, is completely your choice. Just try to get the balls into the box to the best of your ability. Only pick one ball at a time from the delivery platform. Do not reuse missed or dropped balls. We will have a practice round with these rules to familiarize you with the task. Then one more rule will be added.”</td>
</tr>
<tr>
<td>Practice round</td>
<td>Fixed-time progressive schedule for AB group: 1 ball per delivery interval, moving from 2-12 and 12-2 seconds with no break. Fixed-time progressive schedule for BA group: 1 ball per delivery interval, moving from 2-12 and 12-2 seconds with no break.</td>
</tr>
<tr>
<td>Received instructions</td>
<td>“Do not allow the balls to stack at the delivery platform. If you see two balls at the platform, say ‘stop’. When you say stop, I will restart the series. We will redo the series until you can make it through without the balls stacking.”</td>
</tr>
<tr>
<td>Condition A</td>
<td>Fixed-time progressive schedule: 4 balls per delivery interval, moving from 12-2 seconds with no break.</td>
</tr>
<tr>
<td>Condition B</td>
<td>Fixed-time progressive schedule: 4 balls per delivery interval, moving from 2-12 seconds with no break.</td>
</tr>
</tbody>
</table>
**Table 2**

*Displays coding conventions and all dependent measurements.*

<table>
<thead>
<tr>
<th>Coded Data</th>
<th>Variables</th>
<th>Higher-order variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grab: The participant’s hand envelops a new ball on the delivery platform.</td>
<td>Movement interval: grab-to-return time</td>
<td>$\pi_{\text{movement}} = \frac{\text{movement interval}}{\text{delivery interval}}$</td>
</tr>
<tr>
<td>Release: The participant lets go of the ball.</td>
<td>Throw interval: grab-to-release time</td>
<td>$\pi_{\text{throw}} = \frac{\text{throw interval}}{\text{return interval}}$</td>
</tr>
<tr>
<td>Return: The participant sets one foot entirely down within one meter of the delivery platform after having let go of the ball.</td>
<td>Return interval: release-to-grab time</td>
<td>Velocity = $\frac{\text{distance moved} \cdot 2}{\text{movement interval}}$</td>
</tr>
<tr>
<td>Distance: The participant’s frontal foot position on the floor tape at the moment of letting the ball go.</td>
<td>Distance moved: the meters between the participant and the wooden box</td>
<td>Ideal velocity = $\frac{(\text{distance moved} \cdot 2)}{(\text{delivery interval} \cdot \pi_{\text{movement}})}$</td>
</tr>
</tbody>
</table>
Table 3

Displays the number of re-runs participants had for each condition and total session time in minutes.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Condition A</th>
<th>Condition B</th>
<th>Session time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>0</td>
<td>43</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
<td>31</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>3</td>
<td>28</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>38</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>4</td>
<td>26</td>
</tr>
</tbody>
</table>
Figure 1. This image illustrates the experimental setting as described in the method section.
Figure 2. Displays $\Pi_{\text{movement}}$ data as averages per delivery interval for AB participants on the left and BA on the right. The dotted lines are the average $\Pi_{\text{movement}}$ for the participant’s entire session and used to indicate stability. Slow-to-fast titles change from 12-2 second delivery intervals, and fast-to-slow titles change from 2-12 second delivery intervals.
Figure 3. Displays $\Pi_{\text{movement}}$ as AB and BA group averages for each delivery interval. Standard deviation error bars were derived from the average individual performance per delivery interval. The dotted lines are the average $\Pi_{\text{movement}}$ for the entire group and used to indicate stability. Slow-to-fast titles change from 12-2 second delivery intervals, and fast-to-slow titles change from 2-12 second delivery intervals.
Figure 4. Display individual performance for $\Pi_{\text{throw}}$ as average performance for every four trials and AB and BA individual performance respectively. The dotted lines are the average of $\Pi_{\text{throw}}$ for the participant’s entire session and used to indicate stability. Slow-to-fast titles change from 12-2 second delivery intervals, and fast-to-slow titles change from 2-12 second delivery intervals.
Figure 5. Displays $\Pi_{\text{throw}}$ for AB and BA as group averages. Each data point is the average of all individual performance on respective delivery intervals. Standard deviation bars were derived from the average individual performance for the number of trials per delivery interval. The dotted lines are the average $\Pi_{\text{throw}}$ for the entire group and used to indicate stability. Slow-to-fast titles change from 12-2 second delivery intervals, and fast-to-slow titles change from 2-12 second delivery intervals.
Figure 6. Displays individual performance for absolute distance moved towards the box in relation to the delivery intervals. Each data point is the average distance moved for every four trials, or the total number of trials per delivery interval. Slow-to-fast titles change from 12-2 second delivery intervals, and fast-to-slow titles change from 2-12 second delivery intervals.
Figure 7. Displays the highest and lowest actual velocity for each participant that approximated ideal velocity with more than 90%. The data points were derived from average performance on every four trials or trials per delivery interval. Velocity has been artificially arranged from high to low values to facilitate visual inspection.
Figure 8. Displays the total average of individual performance on their respective experimental sessions for ideal and actual velocity. The dotted line is the average actual velocity for all participants. Actual velocity has been arranged from lowest to highest to facilitate visual analysis. Standard deviation bars were obtained from the average individual performance on every four trials or trials per delivery interval.